Competitive Market Supply, Market Equilibrium, and Comparative Statics

Suppose there are $200{,}000$ identical consumers, each with utility function $U_i = x_i^2 y_i$ and income of $M_i = \$400$ for each consumer. The price of good y is normalized to \$1. There are also $20{,}000$ identical perfectly competitive firms producing good x. Each from has the production function $x_i = 3K^{1/2}L^{1/2}$, $w = \$4$, $r = \$9$, and $\overline{K} = 9$ in the short run for each firm.

First we will derive the short run and long run supply function for each firm.

$$\min 4L + 9 \cdot 9$$

$$s.t. \, x_i = 3(9)^{1/2}L^{1/2}$$

We can solve our constraint for L yielding $L = \frac{x_i^2}{81}$.

$$SRVC = \frac{4x_i^2}{81}.$$

$$MC = \frac{\partial SRVC}{\partial x_i} = \frac{8x_i}{81}$$

Recall in perfect competition $p_e = MC \Rightarrow x_i = \frac{81p}{8}$.

The short run supply function is $x_i = \frac{81p}{8}$.

$$\min 4L + 9(K)$$

$$s.t. \, x = 3(K)^{1/2}L^{1/2}$$

Solving the constrain for L yields $L = \frac{x^2}{9K}$.

Substituting in the constraint our maximization problem becomes

$$\min_{\{K\}} 4\frac{x^2}{9K} + 9(K)$$

FOC: $-4\frac{x^2}{9K^2} + 9 = 0$

$$K = \frac{2x}{9}$$

$$L = \frac{x}{2}$$

$$LRTC = 2x + 2x = 4x$$

$$LRMC = 4 = p$$

Demand is perfectly elastic at $p = \$4$

Now we can derive the short run and long run market supply function.

To do this we will simply multiply the short run supply function by 20,000.

$$\mathrm{X_S} = 20{,}000x_i = 20{,}000\frac{81p_x}{8}$$

The short run market supply curve is $X_S = 20{,}000\frac{81p_x}{8}$.

The long run supply curve is unchanged at $p = \$4$.

With this we can now find the competitive market equilibrium price and quantity for good x in the short run and long run.

First we need to individual and market demand functions.

$$\max x_i^2 y_i$$

$$s.t. p_x x + y = 400$$

Substituting in the constraint our maximization problem becomes

$$\max_{\mathrm{x_i}} x_i^2(400 - p_x x_i)$$

FOC: $800x_i - 3p_x x_i^2 = 0$

Solving for x_i we obtain $x_i = \frac{800}{3p_x}$

The individual demand function is $x_i = \frac{800}{3p_x}$.

To find the market demand function we will multiply the individual demand function by 200,000.

$$X_D = 200{,}000x_i = 200{,}000\frac{800}{3p_x}.$$

The market demand function is $X_D = 200{,}000\frac{800}{3p_x}$.

The short run equilibrium implies $X_D = X_S \Rightarrow 200{,}000\frac{800}{3p_x} = 20{,}000\frac{81p_x}{8}$.

Solving for p_x we obtain $p_x = \left(\frac{800(8)}{81(3)}\right)^{1/2} = 16.29$

Substituting this into the market supply function we find $X_e = 32{,}863{,}353.45$

In the long run we know $p_x = \$4$.

Substituting this price into the demand function we obtain $X_e = 13{,}333{,}333.33$

Using this we can calculate the short run profits for each firm.

$$SR\pi = \frac{p_x \cdot X_e - 4L - 9K}{20{,}000}$$

$$SR\pi = \frac{16.29(32{,}863{,}353.45) - 81 - 4\frac{32{,}863{,}353.45}{2}}{20{,}000} = 23{,}480.86$$

We can also calculate how much each from will produce and how many firms will be in the industry in the long run.

Long Run Demand $= 13{,}333{,}333.33$

$LR\pi = 4x - 2x - 2x$

Each firm will produce $x_j \in [0, 13{,}333{,}333.33)$

$s.t. \sum_{j=1}^{k} x_j = 13{,}333{,}333.33$, where k is the number of firms

We can now can show how much profits each firm will make in the long run.

$LR\pi = 0$

Now if we suppose the original $20{,}000$ firms were still in the industry in the long run.

Assuming each firm produces: $\frac{13{,}333{,}333.33}{20{,}000} = 666.67$ then $K = \frac{2x}{9} = 148.15$.

Each firm would employ 148.15 units of capital.

Each firms short run supply at the long run equilibrium is $x = 166.67p_x$.

Consider the following demand and supply function $X_d = 1000 - 10p_x$ and $X_s = 10 + 200p_x$.

The original equilibrium price and quantity occur where $X_d = X_s$.

$1000 - 10p_x = 10 + 200p_x$

Solving for p_x yields $p_e = \frac{990}{210} = \frac{99}{21} \approx 4.71$

Substituting in for p_x yields $X_e = 1000 - \frac{990}{19} \approx 952.85$

The equilibrium price and quantity are $p_e \approx 4.71$ and $X_e \approx 952.85$.

Suppose there is a $1 per unit tax imposed.

The inverse supply is $\frac{X_s}{200} - \frac{1}{20} = p_x$.

With $1 tax the inverse supply becomes $\frac{X_s}{200} + \frac{19}{20} = p_x$

The inverse demand is $100 - \frac{X_d}{10} = p_x$.

The new equilibrium price and quantity occur where $X_d = X_s$.

$\frac{X_{tax}}{200} + \frac{19}{20} = 100 - \frac{X_{tax}}{10}$

Solving for X_{tax} yields $X_{tax} = \frac{\left(\frac{2000}{20} - \frac{19}{20}\right)200}{21} \approx 943.33$

Substituting this into the demand function yields the buyers price $p_b = 100 - \frac{\frac{\left(\frac{2000}{20} - \frac{19}{20}\right)200}{21}}{10} \approx 5.67$

The sellers price will be $1 less than the buyers price: $p_s = p_b - tax \approx 4.67$

With the tax $p_b \approx 5.67$, $p_s \approx 4.67$, and $X_{tax} \approx 943.33$.

We can show that in the short run the tax burden is shared by the firms and the consumers.

$Consumer\ tax\ burden = \frac{(5.67 - 4.71)}{1} \times 100 = 96\%$

$Producer\ tax\ burden = \frac{(4.71 - 4.67)}{1} \times 100 = 4\%$

However in the long run firms will need to make zero profit so the price and quantity will change.

$LR\ P_s = P_e \Longrightarrow P_b = P_e + tax = 5.71$ this will allow firms to make a zero economic profit.

Substituting the buyers price back into the demand yields $X_{tax} = 942.85$.

The long run quantity will be $X_{tax} = 942.85$.

The dead weight loss in the short run is $DWL = \frac{1}{2} \times \$1 \times (952.85 - 943.33) = 4.76$.

The dead weight loss in the long run is $DWL = \frac{1}{2} \times \$1 \times (952.85 - 952.85) = 5$.