Consumer Preference Theory

Axioms:

1) Completeness
2) Reflexive
3) Transitive
4) Continuous
5) Local Non-Satiation
6) Diminishing Marginal Rates of Substitution

Show that the utility function $U(x, y) = xy^2, \forall\ x, y\ \geq 0$ satisfies all axioms

Axioms 1-4 are fundamental properties of real numbers.

The fact that this utility is a function implies axioms 1-4 are satisfied.

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For Axiom 5 to be satisfied we must show that marginal utility is increasing.

$MU_x = \frac{\partial U(x,y)}{\partial x} = y^2$ which is greater than or equal to zero given our constraints on x and y.

$MU_y = \frac{\partial U(x,y)}{\partial y} = 2xy$ which is also greater than or equal to zero given our constraints on x and y.

Axiom 5 is satisfied.

For Axiom 6 to be satisfied we much show that MRS_{xy} is decreasing in x.

Note: along any given indifference curve utility is constant, by definition of indifference curves, which implies $dU = 0$.

$$dU(x, y) = \frac{\partial U(x,y)}{\partial x} dx + \frac{\partial U(x,y)}{\partial y} dy = 0 \Rightarrow \frac{MU_x}{MU_y} = -\frac{dy}{dx}\bigg|_{dU} = 0 = MRS_{xy}$$

$$MRS_{xy} = \frac{y^2}{2xy} = \frac{y}{2x}$$

$\frac{d\ MRS}{dx} = \frac{\frac{dy}{dx}x - \frac{2dx}{dx}y}{4x^2} = \frac{-\frac{y}{2x}x - 2y}{4x^2} = -\frac{5y}{8x^2}$ which is less than or equal to zero given our constraints on x and y.

Axiom 6 is satisfied.

Assuming $p_x = \$2, p_y = \$4$, and $M = \$100$, we can find the utility maximizing bundle (x, y) for the utility function.

$Max\ xy^2$

$s.t. 2x + 4y = 100$

Our Lagrangian is $\mathcal{L} = xy^2 - \lambda(2x + 4y - 100)$

Kuhn-Tucker Conditions:

1) $y^2 - 2\lambda = 0 \Rightarrow \lambda = \frac{y^2}{2}$
2) $2xy - 4\lambda = 0 \Rightarrow \lambda = \frac{xy}{2}$
3) $2x + 4y - 100 \leq 0$
4) $x, y \geq 0$

Combining (1) and (2) yields $y = x$. Substituting this into (3) gives us $x^*(p_x, p_y, M) = \frac{50}{3}$. Substituting this back into (3) gives us $y(p_x, p_y, M) = \frac{200}{9}$.

The utility maximizing bundle for the utility function $U(x, y) = xy^2$ given $p_x = \$2, p_y = \$4$, and $M = \$100$ is $\left(\frac{50}{3}, \frac{200}{9}\right)$.