Elasticity of Individual and Market Demand Functions

We will continue to evaluate the utility function $U(x,y) = x^4y^4, \forall\, x, y \geq 0$ given p_x, p_y, and M.

Recall we calculated the generalized demand functions as $x^* = \frac{m}{2p_x}$ and $y^* = \frac{m}{2p_y}$.

Own Price Elasticity:

$$\frac{\partial x^*}{\partial p_x}\frac{p_x}{x} = -\frac{m}{2p_x^2}\frac{p_x}{x} = -\frac{m}{2p_x}\frac{1}{x} = -\frac{m}{2p_x}\frac{2p_x}{m} = -1$$

$$\frac{\partial y^*}{\partial p_y}\frac{p_y}{y} = -1$$

Income Elasticity:

$$\frac{\partial x^*}{\partial m}\frac{m}{x} = \frac{1}{2p_x}\frac{m}{x} = \frac{m}{2p_x}\frac{2p_x}{m} = 1$$

$$\frac{\partial y^*}{\partial m}\frac{m}{y} = 1$$

Cross Price Elasticity:

$$\frac{\partial x^*}{\partial p_y}\frac{p_y}{x} = 0$$

$$\frac{\partial y^*}{\partial p_x}\frac{p_x}{x} = 0$$

We now go back to the utility function $\mathrm{U_i} = (\mathrm{x_i})^2\mathrm{y_i}$ with 2000 consumers, each with an income of \$500, and $\mathrm{p_y} = \$1$.

Recall we calculated the demand function as $\mathrm{X}^* = \frac{2{,}000{,}000}{3\mathrm{p_x}}$ and $\mathrm{X}^* = \frac{1000\,\mathrm{m}}{3}$.

Own Price Elasticity:

$$\frac{\partial x^*}{\partial p_x}\frac{p_x}{x} = -\frac{2{,}000{,}000}{3\mathrm{p_{x2}}}\frac{p_x}{x} = -\frac{2{,}000{,}000}{3\mathrm{p_x x}}$$

Income Elasticity:

$$\frac{\partial x^*}{\partial m}\frac{m}{x} = \frac{1000}{3}\frac{m}{x} = \frac{1000\,\mathrm{m}}{3\mathrm{x}}$$

Now we will consider the market demand function $x(p_x) = 40 - 10p_x$.

The inverse demand function is $p_x = 4 - \frac{x(p_x)}{10}$.

Total Revenue is $p_x x = 4x - \frac{x^2}{10}$.

Marginal Revenue is $\frac{\partial p_x x}{\partial x} = 4 - \frac{x}{5}$

Revenue maximization occurs at x for which Marginal Revenue is equal to zero.

$4 - \frac{x}{5} = 0 \Rightarrow x = 20$

$p_x(20) = 2$

We can show that the price elasticity of demand at the revenue maximizing point is equal to -1.

Recall the price elasticity of demand is calculated by $\frac{\partial x^*}{\partial p_x}\frac{p_x}{x}$.

$$\frac{\partial x^*}{\partial p_x}\frac{p_x}{x} = -10 \cdot \frac{2}{20} = -1$$