

Individual Consumer and Market Demand

We can find the income-consumption line for the utility function $U(x, y) = x^4y^4, \forall x, y \geq 0$ given $p_x = \$2$ and $p_y = \$3$.

$$\text{Max } x^4y^4$$

$$\text{s. t. } 2x + 3y = 100$$

Our Lagrangian is $\mathcal{L} = x^4y^4 - \lambda(2x + 3y - M)$

Kuhn-Tucker Conditions:

- 1) $4x^3y^4 - \lambda 2 = 0 \Rightarrow \lambda = \frac{4x^3y^4}{2}$
- 2) $4x^4y^3 - 3\lambda = 0 \Rightarrow \lambda = \frac{4x^4y^3}{3}$
- 3) $2x + 3y - M \leq 0$
- 4) $x, y \geq 0$

Combining (1) and (2) yields $y = \frac{2x}{3}$.

The income-consumption function is $y^*(x^*, \bar{p}_x, \bar{p}_y) = \frac{2x}{3}$

Assuming $p_x = \$2, p_y = \3 , and $M = \$100$, we can find the utility maximizing bundle (x, y) for the utility function.

We can substitute the income-consumption function into equation (3) to obtain the utility maximizing bundle.

$$2x + 3\frac{2x}{3} - 100 = 0 \Rightarrow x^*(p_x, p_y, M) = \frac{50}{2}$$

Substituting this back into the income-consumption function we find that

$$y^*(p_x, p_y, M) = \frac{50}{3}$$

Assuming $p_x = \$4, p_y = \6 , and $M = \$200$, we can infer that the utility maximizing bundle (x, y) for the utility function will not change.

We can find λ as a function of M for the utility function $U(x, y) = x^4y^4, \forall x, y \geq 0$ given p_x, p_y , and M .

$$\text{Max } x^4y^4$$

$$\text{s. t. } p_x x + p_y y = M$$

Our Lagrangian is $\mathcal{L} = x^4y^4 - \lambda(p_x x + p_y y - M)$

Kuhn-Tucker Conditions:

- 1) $4x^3y^4 - \lambda p_x = 0 \Rightarrow \lambda = \frac{4x^3y^4}{p_x}$
- 2) $4x^4y^3 - \lambda p_y = 0 \Rightarrow \lambda = \frac{4x^4y^3}{p_y}$
- 3) $p_x x + p_y y - M \leq 0$
- 4) $x, y \geq 0$

Combining (1) and (2) yields $y = \frac{p_x}{p_y} x$.

Substituting this into (3) yields $x^* = \frac{M}{2p_x}$.

Combining this and the previous result gives us $y^* = \frac{M}{2p_y}$.

Substituting this back into (1) yields $\lambda^* = \frac{4\left(\frac{m}{2p_x}\right)^3\left(\frac{m}{2p_y}\right)^4}{p_x} = \frac{4\left(\frac{m}{2p_x}\right)^4\left(\frac{m}{2p_y}\right)^3}{p_y} = \frac{m^7}{32(p_y p_x)^4}$.

We easily can also show that λ^* can be interpreted as the marginal utility of income.

$$U^* = U|_{x=x^*, y=y^*} = \left(\frac{m}{2p_x}\right)^4 \left(\frac{m}{2p_y}\right)^4 = \frac{m^8}{256(p_y p_x)^4}$$

$$\frac{\partial U^*}{\partial m} = \frac{m^7}{32(p_y p_x)^4}$$

$x^*(p_x, p_y, M)$ and $y^*(p_x, p_y, M)$ are the generalized demand functions. We found these earlier as $x^* = \frac{M}{2p_x}$ and $y^* = \frac{M}{2p_y}$.

We can prove the own price demand ordinary functions are downward sloping.

Recall the vertical axis of a demand graph is price.

$$\frac{\partial x^*}{\partial p_x} = \frac{M}{2p_x} = -\frac{M}{2p_x^2} \leq 0$$

$$\frac{\partial y^*}{\partial p_y} = \frac{M}{2p_y} = -\frac{M}{2p_y^2} \leq 0$$

Lastly we can test whether good x and good y are normal or inferior goods.

Recall the condition for a normal good is that $\frac{\partial x^*}{\partial M} > 0$ and the condition for an inferior good is that $\frac{\partial x^*}{\partial M} < 0$.

$$\frac{\partial x^*}{\partial M} = \frac{1}{2p_x} > 0 \Rightarrow \text{good } x \text{ is a normal good.}$$

$$\frac{\partial y^*}{\partial M} = \frac{1}{2p_y} > 0 \Rightarrow \text{good } y \text{ is a normal good.}$$

Suppose there are 2000 consumers, each with an income of \$500, $p_y = \$1$, and utility function $U_i = (x_i)^2 y_i$.

We can derive the own price market demand function for X.

$$U = x_i^2 y_i, i = 1, 2, \dots, 2000, m = \$500$$

$$\text{Max } x^2 y$$

$$\text{s. t. } 500 = p_x x + y$$

$$\text{Our Lagrangian is } \mathcal{L} = x^2 y + \lambda(500 - p_x x - y)$$

Kuhn-Tucker Conditions:

- 1) $2xy - \lambda p_x = 0 \Rightarrow \lambda = \frac{2xy}{p_x}$
- 2) $x^2 - \lambda = 0 \Rightarrow \lambda = x^2$
- 3) $500 - p_x x - y \leq 0$
- 4) $x, y \geq 0$

$$\text{Combining (1) and (2) yields } y = \frac{p_x x}{2}.$$

$$\text{Substituting this into (3) gives us } x^* = \frac{500}{p_x + \frac{p_x}{2}} = \frac{1000}{3p_x}.$$

Recall there are 2000 consumers so to find X^* we need to multiply x^* by 2000.

$$X^* = \frac{2,000,000}{3p_x}$$

Suppose we now let M vary and set $p_x = \$4$. We can derive a market demand function as a function of the sum of consumer incomes.

$$\text{Max } x^2 y$$

$$\text{s. t. } M = 4x + y$$

Our Lagrangian is $\mathcal{L} = x^2y + \lambda(M - 4x - y)$

Kuhn-Tucker Conditions:

- 1) $2xy - \lambda 4 = 0 \Rightarrow \lambda = \frac{xy}{2}$
- 2) $x^2 - \lambda = 0 \Rightarrow \lambda = x^2$
- 3) $M - 4x - y \leq 0$
- 4) $x, y \geq 0$

Combining (1) and (2) yields $y = 2x$.

Substituting this into (3) gives us $x^* = \frac{m}{6}$

Recall there are 2000 consumers so to find X^* we need to multiply x^* by 2000.

$$X^* = \frac{1000m}{3}$$