Monopoly

A monopolist faces the demand curve $x = 250 - p_x$ and has the production function $x = K^{1/2}L^{1/2}$ where $w = \$1$ and $r = \$4$.

Suppose $K = 25$. We can find the short run profit maximizing price and quantity combination.

The monopolist will want to maximize profits.

$$\max_{x} x(250 - x) - 100 - L$$

$$s.t. x = 5L^{1/2}$$

We can substitute in our constraint to obtain:

$$\max_{x} 250x - x^2 - 100 - \frac{x^2}{25}$$

FOC: $250 - 2x - \frac{2x}{25} = 0$

Solving for x yields $x_M = \frac{3125}{26}$

Substituting this into the demand function will yield $p_m = \frac{3375}{26}$.

In the short run the monopolist will provide $x_M = \frac{3125}{26}$ units at a price of $p_m = \frac{3375}{26}$.

We can use the production function to derive the short run demand for labor.

$$x = 5L^{1/2}$$

Solving for L yields $L = \left(\frac{x}{5}\right)^2$.

The short run demand for labor is $L = \left(\frac{x}{5}\right)^2$.

We can calculate short run profits for this firm.

$$\pi = \frac{3125}{26} \cdot \frac{3375}{26} - 100 - \left(\frac{625}{26}\right)^2 = \frac{5044325}{338}.$$

Short run profits for this monopolist are $\pi = \frac{5044325}{338}$.

If we allow K to vary we can derive the long run profit maximizing price and quantity combination.

$\max 250x - x^2 - 4K - L$

$s.t.\, x = K^{1/2}L^{1/2}$

It can easily be shown the $L = 4K$

Our maximization problem becomes $\max_{x} 250x - x^2 - 4x$

FOC: $246 - 2x = 0 \Rightarrow x_M = 123$

Substituting this into our demand function and solving for p gives us $p_M = 127$.

In the long run the monopolist will provide $x_M = 123$ units at a price of $p_m = 127$.

We can calculate long run profits for this firm.

$\pi = p_M \cdot x_M - 4K - L$

Recall $L = 4K$ and $K = \frac{x}{2}$

$\pi = p_M \cdot x_M - 4x$

$123 \cdot 127 - 4 \cdot 123 = 15129.$

Long run profits for this monopolist are $\pi = 15129$.

We can also calculate the amount of capital and labor this monopolist will hire.

Recall $K = \frac{x}{2}$ and $L = 4K$.

Combining these yields $L = 2x$.

So in the long run this monopolist will hire $K = \frac{123}{2}$ and $L = 246$.

We will now compare these results to the situation where this is a perfectly competitive industry.

Recall that in perfect competition $p_e = MC$.

First we need to find the condition factor demands.

$\min 4K + L$

$s.t.\, x = K^{1/2}L^{1/2}$

Substituting in our constraint the maximization problem becomes

$$\min_{\{K\}} 4K + \frac{x^2}{K}$$

FOC: $4 - \frac{x^2}{K^2} = 0 \Leftrightarrow K = \frac{x}{2}$

Recall $L = 4K$ so $L = 2x$.

$$TC = 4K + L = 2x + 2x = 4x$$

$$MC = \frac{\partial TC}{\partial x} = 4 = p_e$$

Substituting this into the demand function we obtain $x_e = 246$.

If this were a perfectly competitive industry the market price would be $P_e = 4$ and the market quantity would be $x_e = 246$.

In the long run firms in a perfectly competitive industry will earn economics profits equal to zero.

$$\pi = TR - TC = (p_e - ATC)x_e = 0$$

We can also calculate the amount of capital and labor hired under perfect competition.

Recall $K = \frac{x}{2}$ and $L = 4K$.

Combining these yields $L = 2x$.

So in the long run this monopolist will hire $K = 123$ and $L = 492$.

Finally we can calculate the amount of the dead weight loss experienced in this market as a result of the monopolist.

Remember in the case of a monopoly, dead weight loss is the cost to society through a reduction of consumer surplus.

$$DWL = \int_4^{127} (250 - x)\,dx - \int_4^{4} (250 - p_x)\,dp_x - 123[127 - 4]$$

$$= \left[250p_x - \frac{p_x^2}{2}\right]_4^{127} - 123[127 - 4] = 7564.5$$