Production Theory

We will consider the production function $x = \frac{10KL}{K+L}$

The marginal rate of technical substitution is defined as $-\frac{dK}{dL}$.

$$dx = \frac{\partial x}{\partial L}dL + \frac{\partial x}{\partial K}dK = 0 \Rightarrow \frac{dK}{dL} = -\frac{MP_L}{MP_K}$$

$$MP_L = \frac{10K^2}{(K+L)^2}$$

$$MP_K = \frac{10L^2}{(K+L)^2}$$

$$MRTS = \frac{\frac{10K^2}{(K+L)^2}}{\frac{10L^2}{(K+L)^2}} = \frac{K^2}{L^2} \geq 0$$

We need to show that this production function exhibits diminishing marginal rates of technical substitution.

$$\frac{d\ MRTS}{dL} = -\frac{2K^2}{L^3} \leq 0$$

This shows the production function exhibits diminishing marginal rates of technical substitution.

Total Product of Labor is $x(L, \bar{K})$

$$x(L, \bar{K}) = \frac{10\bar{K}L}{\bar{K}+L}$$

Average Product of Labor is $\frac{x(L,\bar{K})}{L}$

$$\frac{x(L,\bar{K})}{L} = \frac{10\bar{K}}{\bar{K}+L}$$

Marginal Product of Labor is $\frac{\partial x(L,\bar{K})}{\partial L}$

$$\frac{\partial x(L,\bar{K})}{\partial L} = 10\left(\frac{\bar{K}}{\bar{K}+L}\right)^2$$

We need to verify the production function exhibits diminishing marginal products of labor.

$$\frac{\partial^2 x(L,\bar{K})}{\partial L^2} = -20\frac{\bar{K}^2}{(\bar{K}+L)^3} \leq 0$$

We can test what type of returns to scale this production function exhibits.

$$f(2K, 2L) = \frac{10(2K)(2L)}{2K+2L} = 2\frac{10KL}{K+L} = 2f(K, L) \Longrightarrow \text{Constant Returns to Scale}$$