Profit Maximization by a Competitive Firm: Supply of Goods and Demand for Inputs

Assuming $r = \$4$, $w = \$1$, and $K = 10$ we can find the short run supply function for the production function $x = \frac{10KL}{K+L}$.

Substiting in and solving for L yields: $L = \frac{10x}{100-x}$.

$$TC = 40 + \frac{10x}{100-x}$$

$$MC = \frac{\partial TC}{\partial x} = \frac{1000}{(100-x)^2} = p$$

Solving for x yields $x = 100 - \left(\frac{1000}{p}\right)^{1/2}$

The short run supply curve is $x = 100 - \left(\frac{1000}{p}\right)^{1/2}$.

Suppose there is a technological change that increases output to $x = \frac{20KL}{K+L}$. We can derive the new short run supply curve.

Substituting in and solving for L yields: $L = \frac{10x}{200-x}$.

$$TC = 40 + \frac{10x}{200-x}$$

$$MC = \frac{\partial TC}{\partial x} = \frac{1000}{(200-x)^2} = p$$

Solving for x yields $x = 200 - \left(\frac{1000}{p}\right)^{1/2}$

The new short run supply curve is $x = 200 - \left(\frac{1000}{p}\right)^{1/2}$.

We can test to see if technological change is neutral or biased.

To do this we need to compare the marginal rates of technical substitution.

We will first consider the production function $x = \frac{10KL}{K+L}$

The marginal rate of technical substitution is defined as $-\frac{dK}{dL}$.

$$MP_L = \frac{10K^2}{(K+L)^2}$$

$$MP_K = \frac{10L^2}{(K+L)^2}$$

$$MRTS = \frac{\frac{10K^2}{(K+L)^2}}{\frac{10L^2}{(K+L)^2}} = \frac{K^2}{L^2}$$

We will now consider the production function $x = \frac{20KL}{K+L}$

$$MP_L = \frac{20K^2}{(K+L)^2}$$

$$MP_K = \frac{20L^2}{(K+L)^2}$$

$$MRTS = \frac{\frac{20K^2}{(K+L)^2}}{\frac{20L^2}{(K+L)^2}} = \frac{K^2}{L^2}$$

Both production functions have the same MRTS so the technological change is neutral.

We can now determine the price at which a firm will shut down production in the short run.

Recall the short run shut down rule is when $p \leq \min AVC$.

$$VC = \frac{10x}{100-x}$$

$$\frac{\partial VC}{\partial x} = \frac{1000}{(100-x)^2} \neq 0 \Longrightarrow \text{no minimum}$$

This production function has only one variable input in the SR which has a diminishing marginal product. This firm will not shut down in the short run. This firm will only exit the industry in the long run if price falls below minimum average total cost.

Continuing to use the production function $x = \frac{10KL}{K+L}$, we will assume wage can vary, $p = \$5$, and $K = 10$.

Total revenue is given by $p \cdot x$

We can substitute in to obtain $TR = \frac{500L}{10+L}$.

Using this we derive the marginal revenue product for labor is $\frac{\partial TR}{\partial L} = \frac{5000}{(10+L)^2}$.

We can also derive the average revenue product for labor by $\frac{TR}{L} = \frac{500}{10+L}$.

Finally we can derive the short run demand for labor.

$Max\ \pi$

$$\pi = \frac{500L}{10+L} - r10 - wL$$

$$\max_{\{L\}} \frac{500L}{10+L} - r10 - wL$$

FOC: $\frac{5000}{(10+L)^2} - w = 0$

Solving for L yields $L = \left(\frac{5000}{w}\right)^{1/2} - 10.$

The short run demand for labor is $L = \left(\frac{5000}{w}\right)^{1/2} - 10.$

Consider the following production function $x = K^{1/4}L^{1/4}m^{1/4}$ and let $w = \$1$, $r = \$4$, $p_m = \$2$, and $\overline{K} = 4$ units in the short run.

We can derive the short run supply function for the firm.

$\min L + 16 + 2m$

$s.t. x = 2L^{1/4}m^{1/4}$

Our Lagrangian is $\mathcal{L} = L + 16 + 2m - \lambda\left(x - 2L^{1/4}m^{1/4}\right)$

Kuhn-Tucker Conditions:

1) $1 + \frac{1}{2}\lambda L^{-3/4}m^{1/4} = 0 \Rightarrow \lambda = -2L^{3/4}m^{-1/4}$
2) $2 + \frac{1}{2}\lambda L^{1/4}m^{-3/4} = 0 \Rightarrow \lambda = -L^{-1/4}m^{3/4}$
3) $x - 2L^{1/4}m^{1/4} \leq 0$
4) $L, m \geq 0$

Combining (1) and (2) yields $m = 2L$.

Substituting this into (3) and solving for L gives us that $L = \frac{x^2}{2^{1/2}}$.

Substituting this back in we find $m = \frac{x^2}{2^{3/2}}$.

$$SRVC = \frac{x^2}{2^{1/2}} + 2\frac{x^2}{2^{3/2}}$$

$$MC = \frac{\partial SRVC}{\partial x} = \frac{2x}{2^{1/2}} + \frac{2^2x}{2^{3/2}} = p$$

Solving for x yields $x = \frac{p}{2^{3/2}}$.

The short run supply curve is $x = \frac{p}{2^{3/2}}$.