Translinear squarer derivation

First we write a Kirchoff's law voltage loop equation where all the voltage drops in a closed loop sum to zero. The loop travels the four b-e junctions of the transistors, two of them backwards (so negative) and two forward. Looking at the schematic, follow this path: Into the emitter of Q3 and out its base, then into the emitter of Q1 and out its base, then into the base of Q2 and out its emitter and finally into the base of Q4 and out its emitter, which returns us to the starting point or node.

The V_{BE} voltage of the transistors need to be related to their emitter currents by the Ebers-Moll equation where

$$I_E = I_s(e^{\left(\frac{qV}{kT}\right)} - 1)$$

Where V is the base-emitter voltage, V_{BE}. For matched transistors at the same temperature, q/kT is a constant (about 1/25.3 mV) and saturation current I_S is also a constant, that is, the same for all four transistors. Finally, in the region of operation we are considering, the exponential term is much, much larger than 1, so the -1 constant can be eliminated with negligible effect on accuracy to simplify the math.

The equation indicates that emitter current is controlled by V_{BE}. In our case where currents are controlled by current sources, the equation can be rewritten to express V_{BE} in terms of the current.

To simplify, the symbol V_T is used for the constant kT/q. Eliminating the -1 constant as mentioned earlier, divide both sides of the equation by Is, then take the natural log of both sides and solve for Vbe giving

$$V_{BE} = V_T \ln\left(\frac{I_E}{I_S}\right)$$

Now remember that the currents through Q1 and Q3 we've called I_Y, that through Q4 is I_X, and that through Q2 is I_U. So we're ready to write the equation for voltages around the loop as described initially.

$$-V_T \ln\left(\frac{I_Y}{I_S}\right) - V_T \ln\left(\frac{I_Y}{I_S}\right) + V_T \ln\left(\frac{I_U}{I_S}\right) + V_T \ln\left(\frac{I_X}{I_S}\right) = 0$$

Remembering rules for logs of powers and products and canceling the common V_T, this can be simplified to

$$\ln\left(\frac{I_Y}{I_S}\right)^2 = \ln\left(\frac{I_U I_X}{I_S^2}\right)$$

Then taking the exponential of both sides and canceling the common Is^2 term, we get

$$I_Y^2 = I_X I_U$$

NRK 4/29/2010